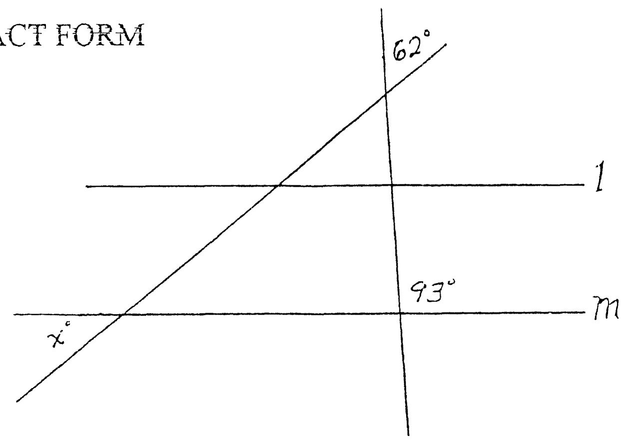
December 5, 2001 WOCOMAL VARSITY MEET

ROUND I: Parallel lines and polygons

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Lines l and m are parallel.
   Find angle measure x.



2. Given regular octagon ABCDEFGH, find the measure of angle ABD.

3. Triangle ABC has sides AC = 5, AB = 12, and BC = 8. The bisector of the exterior angles at B intersects line AC at point D. A line through C parallel to line BD intersects segment AB at point E. Find length AE.

ANSWERS

1. (1 pt) ________________

2. (2 pts) ________________

3. (3 pts) ________________

Bromfield, Hudson, St.Peter-Marian

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ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The letters M, A, T, H represent digits, not necessarily distinct. What digit does A represent if $\sqrt{MATH} = 2^5$, where MATH represents a 4-digit number?

2. An apple orchard has 40 trees and the average yield is 300 apples per tree. For each additional tree planted, the average yield per tree is reduced by 15 apples, for all the trees. If y is the number of new trees planted, write a formula for the total number of apples produced, T.

3. In the magic square shown, the sums of the numbers in each row, column, and both diagonals are the same. Five of these numbers are represented by the letters shown. Find the numerical value of v.

v	24	w
18	x	y
25	z	21

ANSWERS

1. (1 pt) ____________

2. (2 pts) T = ______________________________

3. (3 pts) ____________

Auburn, Burncoat, Shrewsbury

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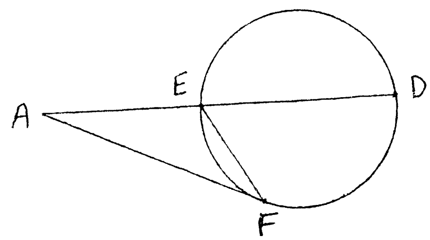
ROUND III: Circles

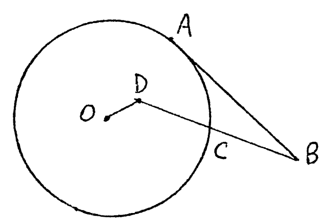
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the radius of a circle in which a central angle of 20^o intercepts an arc of length 2π cm.

2. Segment AF is a tangent AE = EF The measure of arc FD = 100^0. Find the measure of angle FAE.



3. Segment AB is tangent to circle O at point A. Point D is inside the circle and segment DB intersects the circle at C. If BC = DC = 3, OD = 2, and AB = 6, find the radius of the circle.



ANSWERS

1. (1 pt) ____________________ cm

2. (2 pts) ____________________

3. (3 pts) ____________________

Shepherd Hill, Tahanto, Tantasqua

December 5, 2001 WOCOMAL VARSITY MEET

ROUND IV: Sequences and series

NO CALCULATOR USE

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. An auditorium has 15 rows with 20 seats in the first row and 2 more seats in each row thereafter. How many seats are there in the last row?

2. How far has a ball traveled if it is dropped from a height of 8 feet and it bounces 6 times, reaching $\frac{1}{2}$ of its height each time and is caught at the top of that last bounce?

3. Consider the sum $S = 12 + 14 + 16 + 18 + \ldots + N$.
   What is the first value of N that mekes S a perfect square?

ANSWERS

1. (1 pt) ______________

2. (2 pts) ______________ feet

3. (3 pts) ______________

Mass. Academy, Shrewsbury, Southbridge

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ROUND V: Matrices and systems of equations

NO CALCULATOR USE

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find the solutions of the system $\begin{cases} y = x+2 \\ y = x^2 \end{cases}$. Answer in ordered pair (x,y) form.

2. Find the sum $x + y + z$ if

$$\begin{bmatrix} xa+y \\ x \\ ya+z \end{bmatrix} = \begin{bmatrix} 2 & a & 3 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ a \end{bmatrix}$$

3. There are four unequal positive integers a, b, c, and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$?

ANSWERS

1. (1 pt) ______________________________

2. (2 pts) ________________

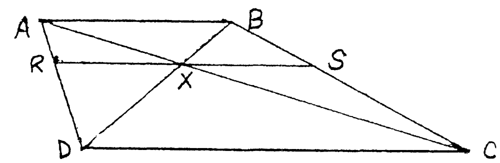
3. (3 pts) ________________

Doherty, Hudson, Worcester Academy

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM and ON THE SEPARATE TEAM ROUND ANSWER SHEET 2 points each

1. ABCD is a trapezoid with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at X. $\overline{RS}$ is parallel to $\overline{DC}$. If AB = 5 and DC = 15, find length RS.



2. Brad is 24 years old. Brad is twice as old as Erin was when Brad was as old as Erin is now. How old is Erin?

3. The diameter of a circle has length 15 and is divided into five equal segments. Find the total length of the four chords drawn perpendicular to the diameter at the points of division.

4. Brianne produces a sequence of positive integers by following three rules. She starts with a positive integer, then applies the appropriate rule to the result, and continues in this fashion.
   Rule 1: If the integer is less than 10, multiply it by 9.
   Rule 2: If it is even and greater than 9, divide it by 2.
   Rule 3: If it is odd and greater than 9, subtract 5 from it.
   A sample sequence: 23, 18, 9, 81, 76, . . .
   Find the 98th term of the sequence that begins with 98.

5. If $\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x-1}=\dfrac{3x^2+1}{x^3-x}$, determine the numerical value of B.

6. The number h is called the harmonic mean of a and b if $\dfrac{a-h}{h-b}=\dfrac{a}{b}$.
   Solve this equation for h.

7. How many sets of 3 consecutive odd integers have a sum between 26 and 52?

8. If $4^x - 4^{x-1} = 48$, find $(2x)^x$.

9. If m:n = 3:4 and r:s = 1:3, find the value of $\dfrac{4mr-ns}{2ns-mr}$.

December 5, 2001 WOCOMAL Varsity Meet ANSWERS

ROUND I ∥ lines polygons	1. 1 pt	31°	may omit degrees symbol
	2. 2 pts	112.5°	$112\frac{1}{2}$ OK
	3. 3 pts	4	

ROUND II alg 1	1. 1 pt	0 or $A=0$	
	2. 2 pts	$T = (40+y)(300-15y)$ or $12000 - 300y - 15y^2$ or $15(800-20y-y^2)$	$15(20-y)$ ok
	3. 3 pts	23	

ROUND III circles	1. 1 pt	18 cm	
	2. 2 pts	25°	
	3. 3 pts	$\sqrt{22}$	

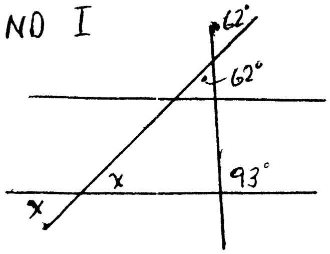
ROUND IV seq series	1. 1 pt	48	
	2. 2 pts	$23\frac{5}{8}$ or 23.625 ft	
	3. 3 pts	60	

ROUND V mat sys	1. 1 pt	$(-1,1)$, $(2,4)$	need both
	2. 2 pts	10	
	3. 3 pts	33	

TEAM ROUND 2 pts each

1. 7.5 or $7\frac{1}{2}$ or $\frac{15}{2}$
2. 18 years
3. $24 + 12\sqrt{6}$ or $12(2+\sqrt{6})$
4. 27
5. 2
6. $\dfrac{2ab}{a+b}$
7. 5
8. 216
9. 0

ROUND I

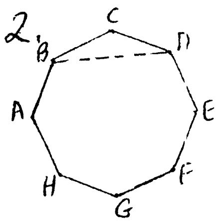
1. 

ext. ∠ of △

$x + 62° = 93°$

$x = 31°$

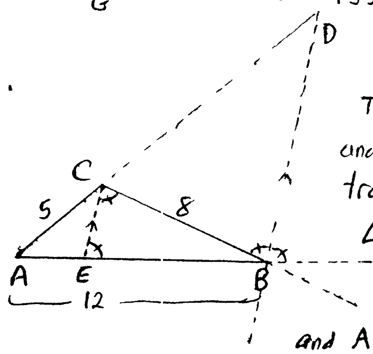
$\ell \parallel m$ is irrelevant

2. 

$m\angle C = 135°$ $\left(180° - \frac{360°}{8}\right)$

Other two ∠'s in isos △BCD are $\frac{45}{2} = 22\frac{1}{2}°$ each

$m\angle ABD = 135° - 22\frac{1}{2}° = 112\frac{1}{2}$

3. 

The bisected ∠ at B and the ∥ lines and transversals make $\angle BCE \cong \angle BEC$.

$\therefore BE = BC = 8$

and $AE = 12 - 8 = 4$

ROUND II

1. $\sqrt{MATH} = 2^5 \Rightarrow MATH = 2^{10} = 1024$
$\therefore A = 0$

2. $T = (\text{new \# trees})(\text{new ave yield})$
$T = (40 + y)(300 - 15y)$

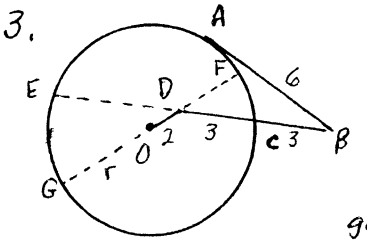
3. Since v appears in 1st row, 1st column, and one diagonal, $24 + w = 18 + 25 = x + 21$. $\therefore w = 19$ and $x = 22$. Then 25, 22, and 19 are on a diagonal and the "magic sum" is 66. $v + 43 = 66 \Rightarrow v = 23$.

ROUND III

1. $2\pi = \frac{20}{360}(2\pi r) \Rightarrow r = 18$ cm

2. $AE = EF$ makes $m\angle A = m\angle EFA = x$.
$x = \frac{1}{2}\left(m\widehat{FD} - m\widehat{EF}\right) = \frac{1}{2}(100° - 2x)$
$x = 50 - x \Rightarrow x = m\angle FAE = 25°$

ROUND III cont.

3. 

Extend $\overline{OD}$ and $\overline{BD}$ as shown to F, G, and E

$BC \cdot BE = BA^2$

$3(6 + DE) = 6^2$

gets $DE = 6$.

Next $DE \cdot DC = DF \cdot DG$

$6 \cdot 3 = (r-2)(r+2)$

$18 = r^2 - 4$ and $r = \sqrt{22}$

ROUND IV

1. Arith seq. $t_{15} = t_1 + 14d$
$= 20 + 14 \cdot 2 = 48$
OR count by 2's from 20

2. Six "downs" starting with 8 and six "ups" starting with 4.
$8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$
$+ 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 23\frac{5}{8}$ ft

3. Let N be the nth term.
$N = 12 + 2(n-1)$
$S = \left(\frac{12+N}{2}\right)n = \left(\frac{12 + 12 + 2(n-1)}{2}\right)n$
$= \ldots = (n+11)n$ is to be a perfect square. Trying $n = 4, 9, 16, 25$ or maybe something more clever finds $n = 25$ as the smallest positive n making a perfect square, $36 \cdot 25$. Then $N = 12 + 2 \cdot 24 = 60$.

ROUND V

1. Equating the y's, $x^2 = x + 2$.
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2$ or -1
Using $y = x^2$ we get the (x, y) pairs
$(2, 4)$ and $(-1, 1)$

ROUND V

2. $$\begin{bmatrix} 2 & a & 3 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ a \end{bmatrix} = \underbrace{\begin{bmatrix} 2+5a \\ 5 \\ 3+2a \end{bmatrix} = \begin{bmatrix} xa+y \\ x \\ ya+z \end{bmatrix}}_{\text{gets } x=5,\ y=2,\ z=3}$$

and $x+y+z=10$

3. $N = 5a+3b+5c$ ← mult by 4
$N = 4a+5b+4c$ ← mult by 5

$4N = 20a+12b+20c$
$5N = 20a+25b+20c$ } Then subtract

$-N = -13b$ or $N = 13b$. Also $131 < N < 150$

$\therefore\ 131 < 13b < 150$ and b is a pos int.

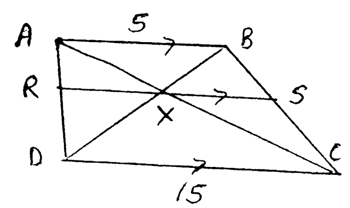
$\therefore\ b = 11$ and $N = 143$.

From original eq, $143 = 5a+33+5c$

$110 = 5a+5c$ and $a+c=22$.

Finally $a+b+c = 33$

TEAM ROUND



1. $\triangle AXB \sim \triangle CXD$
side ratio 1:3

$\triangle ARX \sim \triangle ADC$
side ratio 1:4

$RX = \frac{1}{4}DC = \frac{15}{4}$

$\triangle XSC \sim \triangle ABC$, side ratio 3:4

$XS = \frac{3}{4}AB = \frac{15}{4}$

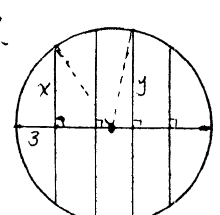
$RS = RX + XS = \frac{15}{2}$

2.

	now	then
Brad	24	x
Erin	x	12

$24 - x = x - 12$
$\vdots$
$x = 18$

3.



$x^2 + 4.5^2 = 7.5^2$
gets $x = 6$

$y^2 + 1.5^2 = 7.5^2$
gets $y = 3\sqrt{6}$

$4x + 4y = 24 + 12\sqrt{6}$

4. 98, 49, 44, 22, 11, 6, 54, 27, 22, 11, 6, etc
1st (98); 5th, 95th (11); 98th (27); repeats; 10th (11)

5. Since $x(x+1)(x-1) = x^3 - x$, mult by it to get

$A(x+1)(x-1) + Bx(x-1) + Cx(x+1) = 3x^2+1$
$\vdots$
$x^2(A+B+C) + x(-B+C) + (-A) = 3x^2+1$

$$\left.\begin{cases} A+B+C = 3 \\ -B+C = 0 \\ -A = 1 \end{cases}\right\} \Rightarrow A=-1,\ B=2,\ C=2$$

6. $\frac{a-h}{h-b} = \frac{a}{b} \Rightarrow ab - bh = ah - ab$

$2ab = ah + bh$ and $h = \frac{2ab}{a+b}$

7. Sum = 3 times middle one so possible middle ones are 9, 11, 13, 15, 17
Ans: 5

8. $4^x - 4^{x-1} = 48$
$4^{x-1}(4-1) = 48$
$4^{x-1} = 16 \Rightarrow x-1 = 2$ and $x = 3$
$(2x)^x = 6^3 = 216$

9. Just use $m=3, n=4, r=1, s=3$.

$\frac{4mr - ns}{2ns - mr} = \frac{12-12}{24-3} = 0$